

$$a \cdot b = b \cdot a$$

$$1 = \frac{1}{1}$$

$$a = \frac{a}{1}$$

$$a \cdot 1 = a$$

$$\frac{a}{a} = 1$$

Fractions

practice also in reverse,
i.e. read from right to left.

multiplication

ex. 1:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

ex. 2:

$$a \cdot \frac{b}{c} = \frac{a \cdot b}{1 \cdot c} = \frac{a \cdot b}{1 \cdot c} = \frac{a \cdot b}{c} = \frac{a b}{c}$$

division

ex. 3:

$$\frac{a}{\frac{b}{c}} = a * \frac{c}{b} = \frac{a}{1} * \frac{c}{b} = \frac{a * c}{1 * b} = \frac{ac}{b}$$

no change

the fraction line becomes multiplication

this fraction turns upside-down

ex. 4:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} * \frac{d}{c} = \frac{a * d}{b * c} = \frac{ad}{bc}$$

no change

turn upside-down

ex. 5:

$$\frac{1}{\frac{d}{b}} = 1 * \frac{b}{d} = \frac{1}{1} * \frac{b}{d} = \frac{1 * b}{1 * d} = \frac{b}{d}$$

no change

turn

Fractions $\swarrow$ ~~multiply~~ amplifying
 $\searrow$ simplifying.

amplify by
 $\frac{a}{b} \times \frac{c}{c}$
 we multiply the
 top & the bottom

$$\frac{a}{b} = \frac{a}{b} = \frac{a \times c}{b \times c} = \frac{a \cdot c}{b \cdot c} = \frac{ac}{bc}$$

this is the same as:

$$\frac{a}{b} = \frac{a \cdot 1}{b \cdot 1} = \frac{a \cdot c}{b \cdot c} = \frac{a \cdot c}{b \cdot c} = \frac{ac}{bc}$$

simplify by
 $\frac{a}{b} \div \frac{c}{c}$
 we divide the
 top & the
 bottom by c

$$\begin{aligned} \frac{a}{b} &= \frac{a}{b} = \frac{a \div c}{b \div c} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{a}{c} \times \frac{c}{b} = \\ &= \frac{a \cdot c}{c \cdot b} = \frac{ac}{bc} \end{aligned}$$

turn upside-down

when we simplify
 this is actually
 what we do:

$$\begin{aligned} \frac{a \cdot c}{b \cdot c} &= \frac{a \cdot \cancel{c}}{b \cdot \cancel{c}} = \frac{a}{b} \cdot \left(\frac{c}{c} \right) = \\ &= \frac{a}{b} \cdot 1 = \frac{a}{b} \end{aligned}$$

$\frac{c}{c} = 1$

Adding & Subtracting Fractions

sort the letters to notice equalities

addition:

$$\frac{a}{b} + \frac{c}{d} = \frac{d}{d} \frac{a}{b} + \frac{b}{b} \frac{c}{d} = \frac{ad}{bd} + \frac{c \cdot b}{d \cdot b} =$$

$\underbrace{d \cdot b}_{= b \cdot d}$

$$= \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{b \cdot d}$$

we can't do it b/c the denominators are different.

↓
solution: multiply by each other's denom.

(you actually need the ~~smallest~~ lowest common denominator: LCD)

now they are the same, it means you can add.

$$\frac{a}{b} - \frac{c}{d} = \frac{d}{d} \frac{a}{b} - \frac{b}{b} \frac{c}{d} = \frac{a \cdot d}{b \cdot d} - \frac{c \cdot b}{d \cdot b} =$$

sort alphabetically

Subtraction

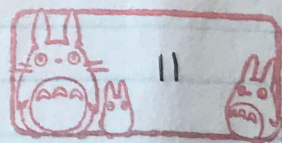
is similar, we can only do it after the denominators (bottoms) are the same

$$= \frac{a \cdot d}{b \cdot d} - \frac{b \cdot c}{b \cdot d} = \frac{a \cdot d - b \cdot c}{b \cdot d} =$$

the same denominator

$$= \frac{ad - bc}{bd}$$

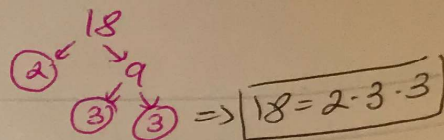
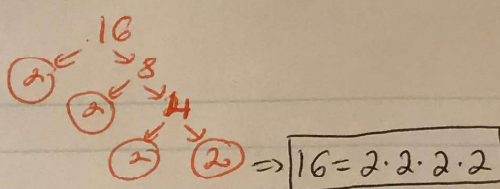
the notation often drops the multiplication sign when using only letters in various equations



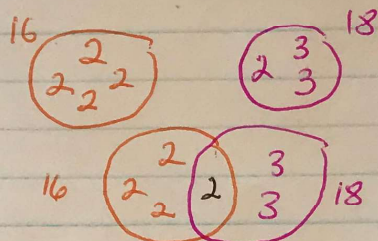
LCD

$\frac{1}{16} + \frac{1}{18} = \frac{18}{16} \cdot \frac{1}{18} + \frac{16}{18} \cdot \frac{1}{16}$ → these wld give us unnecessarily large numbers. Instead use the LCD.

Step 1:
prime
decomposition



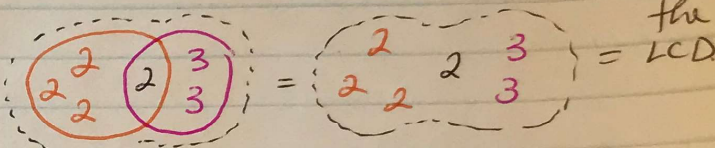
Step 2:
Think Venn diagrams
for the primes



Step 3
Intersect them

only
black = what they hv
in common

Step 4
take each part
only once: the differences & the intersection:



the LCD = $2 * 2 * 2 * 2 * 3 * 3$

The two denominators 16 & 18 must be amplified to become the LCD

$$\frac{1}{16} + \frac{1}{18} = \frac{1}{2 \cdot 2 \cdot 2 \cdot 2} + \frac{1}{2 \cdot 3 \cdot 3}$$

↓
it's missing
3 · 3
↓
it's missing
2 · 2 · 2

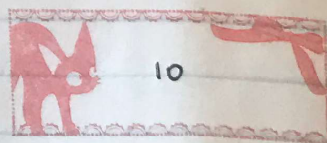
$$= \frac{3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 2} + \frac{2 \cdot 2 \cdot 2}{2 \cdot 3 \cdot 3} =$$

$$= \frac{1 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3} + \frac{1 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2} =$$

these two denominators are the same
so we can proceed & equal 144.

$$= \frac{9}{144} + \frac{8}{144} = \frac{9+8}{144} = \frac{17}{144} \checkmark$$

we are
allowed to add
them b/c they hv the
same denominators



making a decimal point # into a fraction:

ex 1

0.0004

step 1: extract the 4.

1 position

step 2: count the zeros: 3

count the number of positions for (4): 1 more

3
zeros

add them: $3+1=4 \rightarrow 4$ becomes an exponent

used 4
 $4 \rightarrow 10$
to
make an exponent

step 3 divide the number (4) by 10^4

$$\frac{4}{10^4} = \frac{4}{10000}$$

ex 2

0.0036

2
zeros

2 positions

$$2+2=4$$

$$10^4$$

$$\rightarrow \frac{36}{10^4} = \frac{36}{10000}$$

ex 3

0.00108

2 pos

3 pos

$$2+3=5$$

$$\Downarrow \\ 10^5$$

$$\rightarrow \frac{108}{10^5} = \frac{108}{100000}$$