

20. To save for a new highway tractor, a truck company deposits \$11 500 at the end of every 6 months into an account with an annual percentage rate of 5%, compounded semi-annually. Determine the number of deposits needed so that the account has at least \$150 000. Use the formula  $FV = \frac{R[(1+i)^n - 1]}{i}$ , where  $FV$  is the future value,  $n$  is the number of equal periodic payments of  $R$  dollars, and  $i$  is the interest rate per compounding period expressed as a decimal.

11 500 every 6m

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$$150\,000 = \frac{11\,500[(1+0.05)^n - 1]}{0.05}$$

$$7500 = 11\,500(1.05)^n - 11\,500$$

$$19\,000 = 11\,500(1.05)^n$$

$$1.05^n = \frac{38}{23}$$

$$\log_{1.05} \left( \frac{38}{23} \right) = n$$

$$n = 10.29 \dots \leftarrow \text{incorrect}$$

$$\log_a b = c \Leftrightarrow a^c = b$$

Correct:

$$150\,000 = \frac{11\,500(1.025)^n - 11\,500}{0.025} \quad | \quad (\times) 0.025$$

$$\underbrace{150\,000 \times 0.025}_{3750} = 11\,500(1.025)^n - 11\,500 \quad | \quad (+) 11\,500$$

$$15\,250 = 11\,500(1.025)^n \quad | \quad (\div) 11\,500$$

$$1.326087 = 1.025^n \quad \log_a b = c \Rightarrow a^c = b$$

$$\log_{1.025}(1.326087) = n \quad \leftarrow \text{use your calculator}$$

$n = 11.42983 \rightarrow$  this went past the 11th contribution  
but we are only interested in a whole  
number of contributions not partial like 11.4

therefore only after the 12th contribution

will we have an amount  $\geq 150,000$

$\Downarrow$

answer:  $n = 12$  contributions

solved like this  
means every 6 months  
5% are received,  
but that is the  
annual value  
 $\Downarrow$   
semi-annual =  $\frac{5\%}{2} = 2.5\%$   
 $\Downarrow$   
 $i = 0.025$