CH. 9, Unit 9.3, Apply

Ex 10/p.498 Unit 9.3

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(a) to graph we must know the roots of the parabola
∴ solve for a
$L(a)=\emptyset$

$$L(a) = -0.000125a^2 + 0.040a - 2.442 =$$

$$= \frac{-125}{10^6}a^2 + \frac{40}{1000}\cdot a - \frac{2442}{1000}$$

$$= \frac{1}{10^3}\left[\frac{-125}{10^3}a^2 + 40a - 2442\right]$$

$$L(a)=0 \Leftrightarrow$$

$$-\frac{125}{10^3}\cdot a^2 + 40a - 2442 = 0.$$

$$\Delta = b^2 - 4ac$$
$$= 40^2 - 4\left(-\frac{125}{10^3}\right)\cdot(-2442) =$$
$$= 1600 - \frac{1{,}221{,}\cancel{000}}{\cancel{10^3}} = 379$$

19.468

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-40 \pm \sqrt{379}}{2\cdot\left(\frac{-125}{10^3}\right)}$$

$$x_1 = \frac{-40 + 19.468}{2}\cdot\left(-\frac{10^3}{125}\right) = (-0.082128)(-10^3)$$

$$= 82.13 \checkmark$$

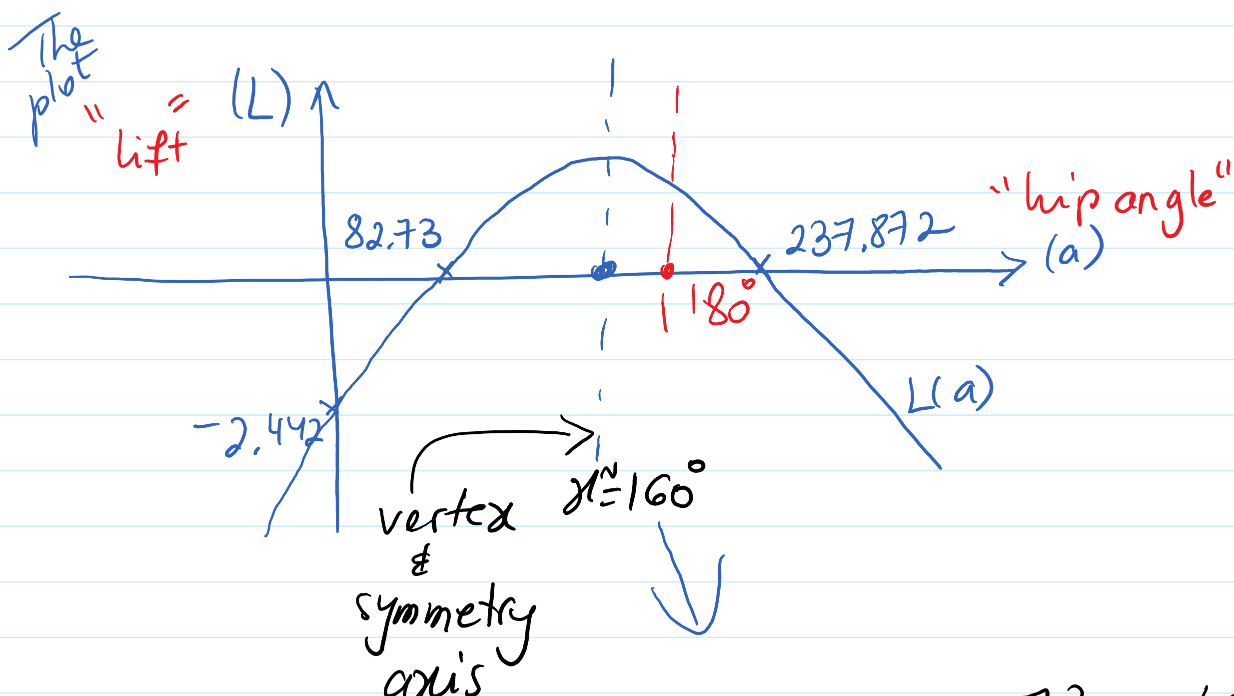
$$x_2 = \frac{-40-19.468}{2} \cdot \left(\frac{-10^3}{125}\right) = -29.734\left(-\frac{10^3}{125}\right) =$$

$$= 0.237\,872 \times 10^3 = 237.872 \checkmark$$

These are the roots of the original parabola $L(a)$

$$L(a) = -0.000125a^2 + 0.040a - 2.442$$

"frowny parabola" $\cap$ → y-int = -2.442



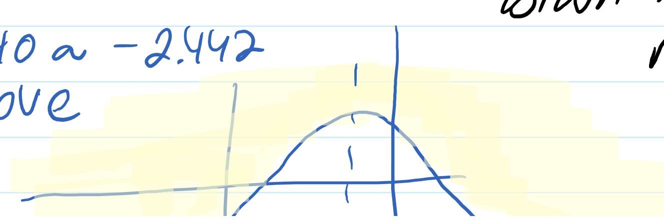
$$82.73 + \frac{237.872 - 82.73}{2} \approx 160°$$

the vertex has to be in the middle btwn the two roots.

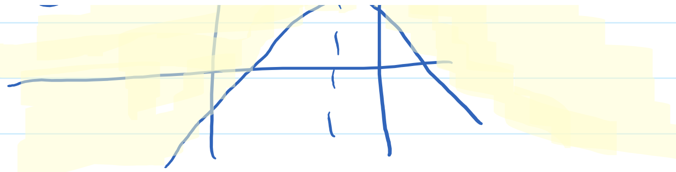
Graphing the inequality

$L \geq -0.000125a^2 + 0.040a - 2.442$

is all the y-s above the curve and



the curve and including the curve as marked by the highlight.



however....

we have to remember this is a real-life problem, so what are the restrictions on L & a?

1) L cannot be negative

2) a can only be btw $[0°, 180°]$ biological limitation on body

we intersect the three things:

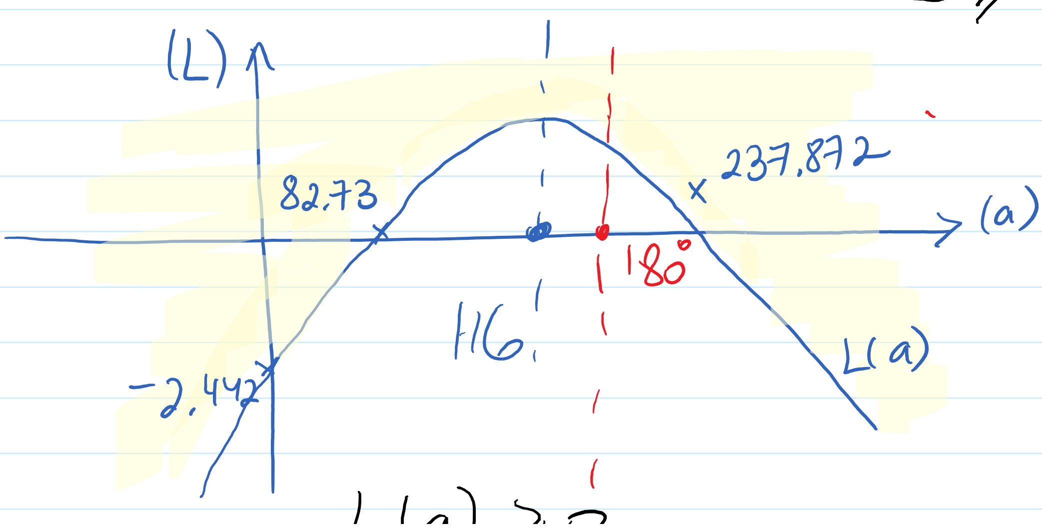
(i) the curve: $L \geq -0.000125a^2 + 0.040a - 2.442$

(ii) $a \in [0, 180]$

(iii) $L \geq 0$

$\boxed{\text{Figure for (i)}}$

$L \geq$ its formula



$L(a) \geq 0$

Figure for ii

$a \in [0, 180^\circ]$

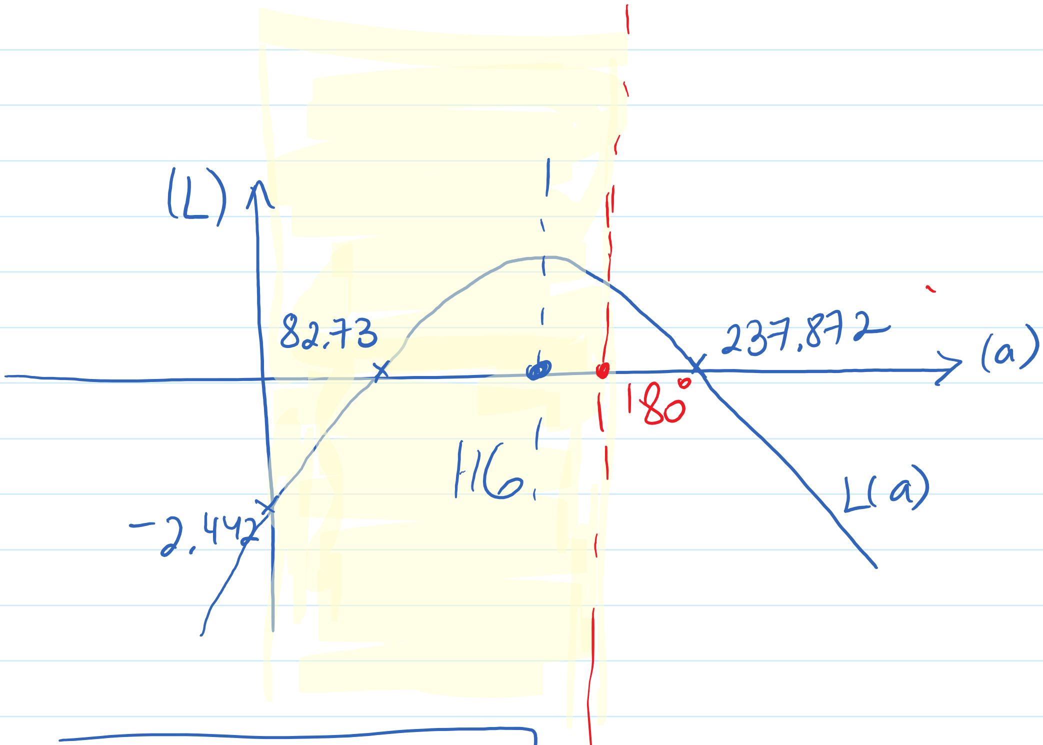
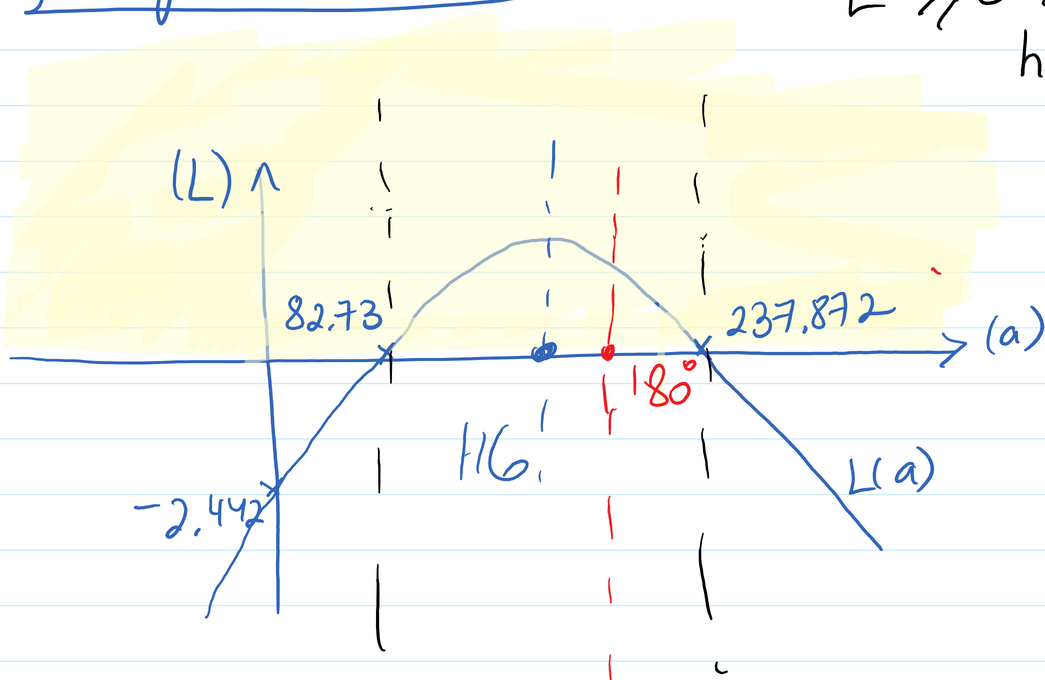


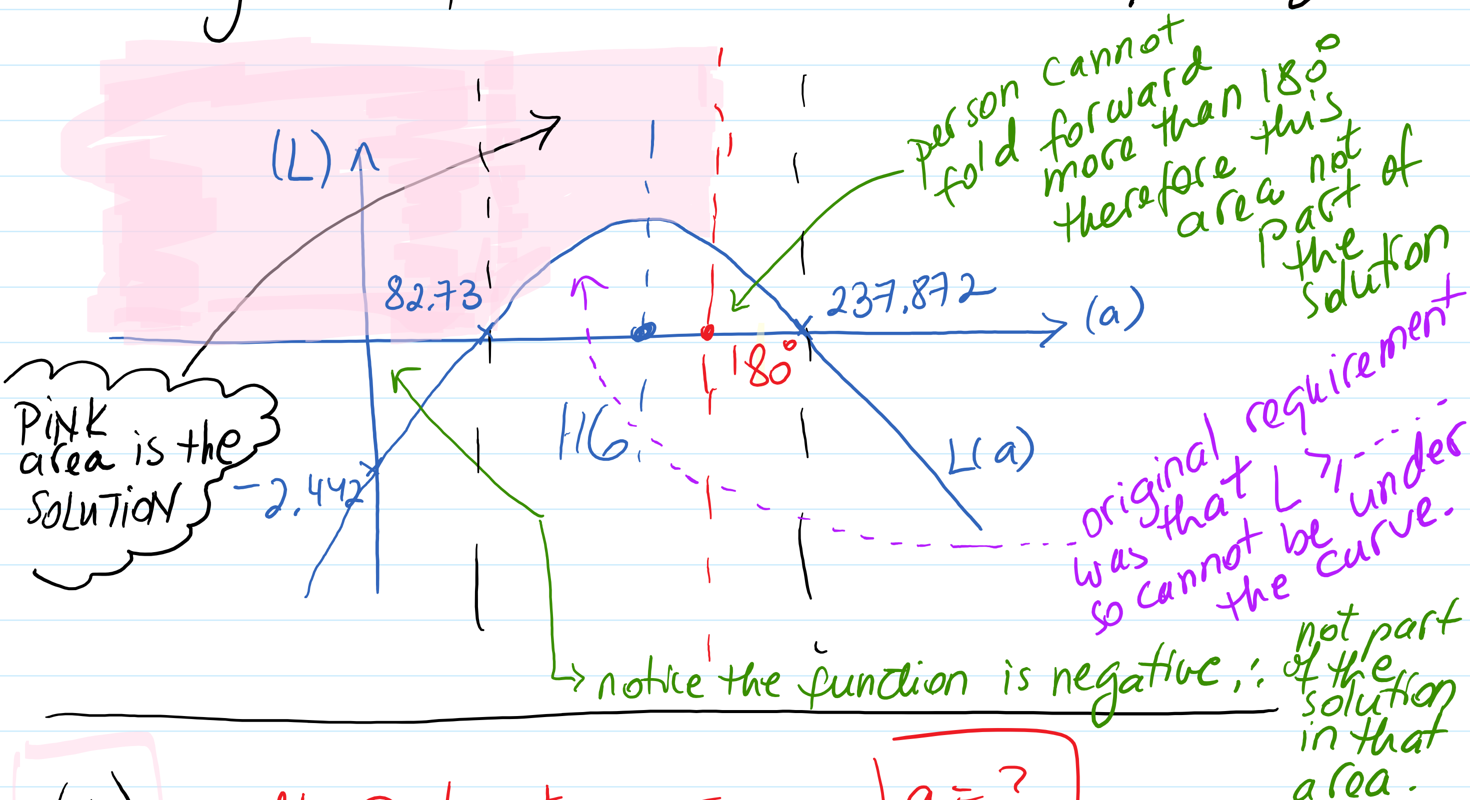
Figure for iii



$L \geq 0$ ← this inequality has nothing to do with the curve, it is a general statement.

when overlapping the 3 figures

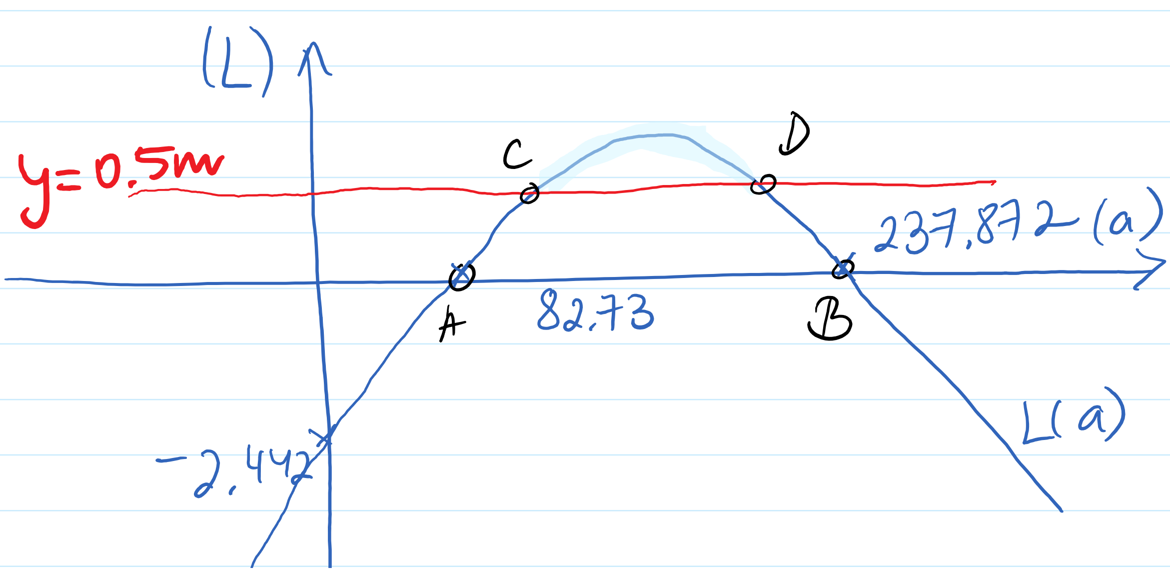
when overlapping the 3 figures we got the follow area for the inequality

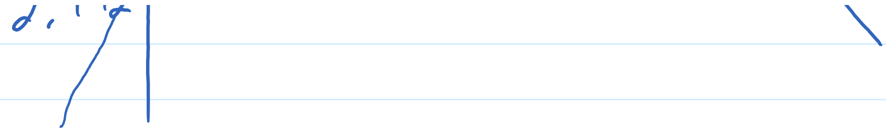


(b) lift @ least 0.5m, $\boxed{a = ?}$

is equivalent to the curve L(a) "being above" the line $y = 0.5$

mathematically: $L(a) \geq 0.5$





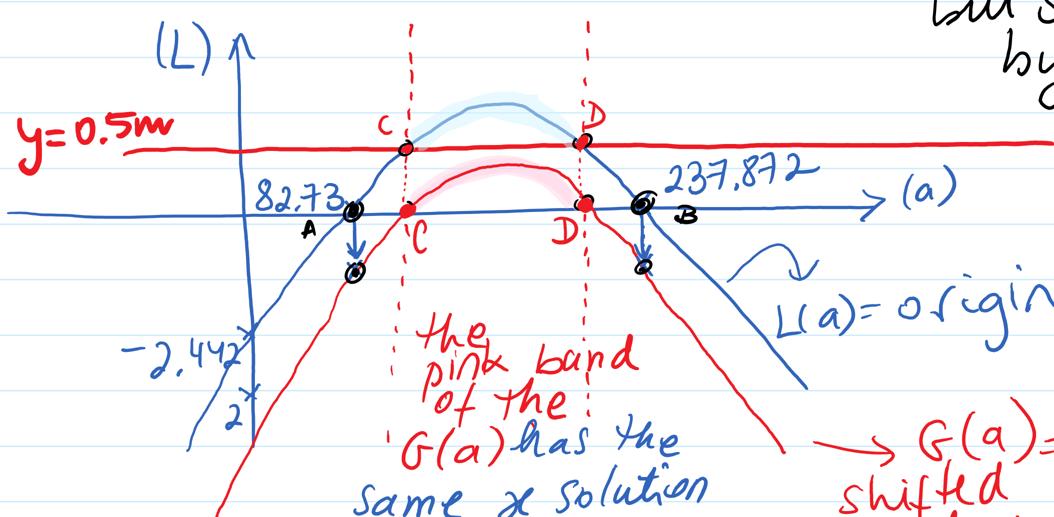
$L(a) \geq 0.5$

$L(a) - 0.5 \geq 0$ & $L(a) = -0.000125a^2 + 0.040a - 2.442$

$\underbrace{L(a) - 0.5}$

redefine then: (in red) $G(a)$ = (in blue) $L(a) - 0.5$

& $G(a) \geq 0$. ← this is like $L(a)$ but shifted down by 0.5 units



the pink band of the $G(a)$ has the same x solution as the blue area on the $L(a)$ function

$L(a)$ = original

→ $G(a) = L(a) - 0.5$ shifted down by 0.5 units.

the solution of $G(a) \geq 0$

is x values between the roots of $G(a)$: C & D.

It is thus important to find the roots of $G(a)$:

$$\Rightarrow \begin{cases} L(a) = -0.000125a^2 + 0.040a - 2.442 \\ G(a) = L(a) - 0.5 \end{cases}$$

$$\Rightarrow \boxed{G(a) = L(a) - 0.5}$$

$$= -0.000125a^2 + 0.040a - 2.442 - 0.5$$

$$= -125\times10^{-6}a^2 + 40\times10^{-3}a - 2442\times10^{-3} - 5\times10^{-1}$$

Solve for a:

$$-125\times10^{-6}a^2 + 40\times10^{-3}a - 2442\times10^{-3} - 5\times10^{-1} = \emptyset \quad \Big| \times \text{ both sides by } 10^6$$

$$-125a^2 + 40\times10^3 a + 2442\times10^3 - 5\times10^5 = \emptyset \quad \Big| \times(-1)$$

$$125a^2 - 40,000a + 2,442,000 - 500,000 = \emptyset$$

$$125a^2 - 40,000a + 2,924,000 = 0$$

$$\Delta = b^2 - 4ac = (-4\times10^4)^2 - 4(125)\cdot2924\cdot10^3$$

$$= 138\times10^6$$

$$x_{1,2} = \frac{-b\pm\sqrt{\Delta}}{2a} = \frac{-(-4\times10^4)\pm\sqrt{138\cdot10^6}}{2\cdot125} =$$

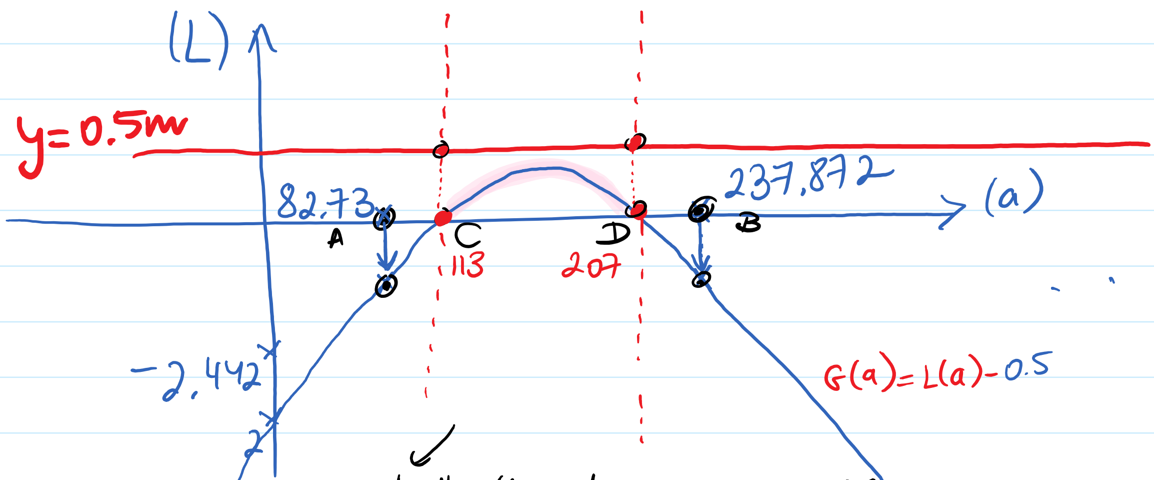
$$= \frac{4\cdot10^4 \pm 11.75\sqrt{10^6}}{250} = \frac{4\cdot10^4 \pm 11.75\times10^3}{250} =$$

$$= \frac{4\cdot10^4 \pm 11750}{250} (10 = \frac{\cancel{10}[4\cdot10^3 \pm 1175]}{25\times\cancel{10}} =$$

$$= \frac{4000\pm1175}{25} \xrightarrow{(+)} \frac{4000+1175}{25} = 207 \equiv \text{point D}$$

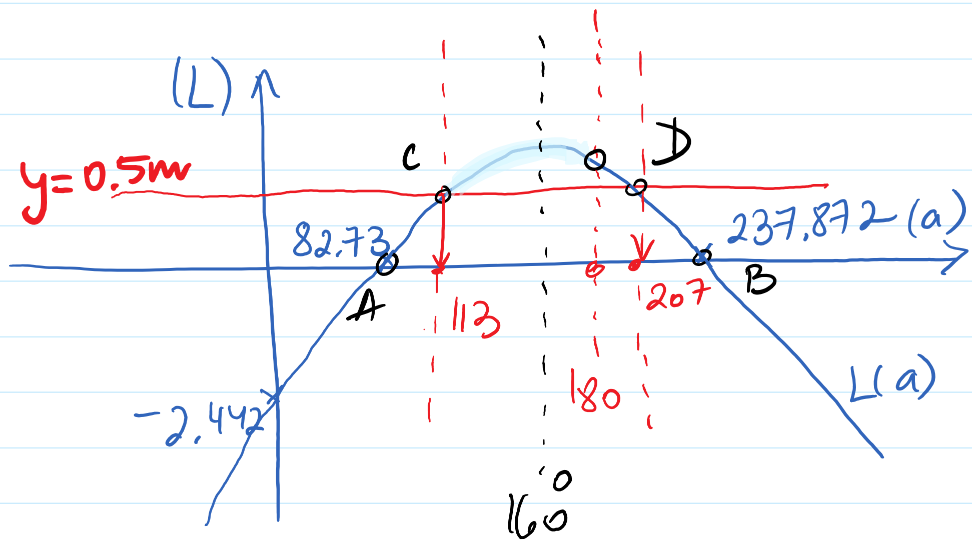
$$\xrightarrow{(-)} \frac{4000-1175}{25} = 113 \equiv \text{point C}$$

$\frac{4800 - \ldots}{25} = 113$ = point C



initially the solution for a is between the roots $113^\circ < a \leq 207^\circ$ however this range must b intersected w/ the physical limitations f a real life problem, a.k.a $a \in [0^\circ, 180^\circ] \equiv 0^\circ \leq a \leq 180^\circ$

∴ the solution is $113^\circ \leq a \leq 180^\circ \equiv a \in [113^\circ, 180^\circ]$.



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