MHR book - p.29/ex 16 "Extend"

Saturday, August 3, 2019 9:53 PM

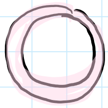
(Alternate solution follows @ th bottom)

How much distance does each circle contributes?

notations: D_i = the distance contribution of a the i-th circle (from the largest to the smallest). Do not confuse with the bottoms, this is the distance contributed by one circle.

c_i = the diameter of the i-th circle.

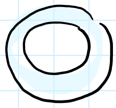
D_1:



the bottom of c_1 is: 20 cm

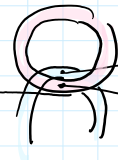
$D_1 = c_1$

D_2:



size: $c_2 = c_1 - 1\,cm = 20 - 1 = 19\,cm$

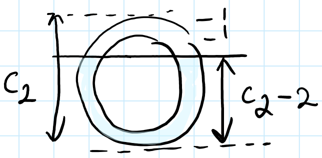
position:



D_1 ← → 1 unit taken by blue

→ 1 unit taken by red

total: −2 units up



D_2 = the distance contribution of c_2

$D_2 = c_2 - 2$

$D_3 = c_3 - 2$

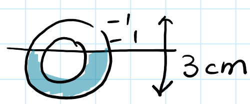
⋮

$D_i = c_i - 2$

⋮

$D_n = 3 - 2 = 1\,cm.$

c_{last}: "the last"



contributions of dist.: $D_1, \ldots D_i, \ldots D_n$

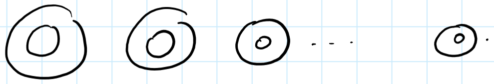
$D_1 = 20$, $D_i = c_i - 2$, $D_n = 1$

Attention: do not confuse the distance contributions of the circles with their respective diameters.

almost looks like we could apply: $S_n = \frac{n}{2}(D_1 + D_n)$ to find the total distance. However, the sequence (& series)

D_n (S_n) contains a subseries c_i, so we can't apply this formula until that has been resolved.

So let's resolve:



20	20-1	20-2	3
c_1	c_2	c_3	c_n

$c_i = 20-(i-1)$ what is n? the level @ which 3cm circle appears.

$c_i = 20-(i-1)$

$c_i = 20-i+1$

$\boxed{c_i = 21-i}$ ← genera formula for diameter of i-th circle.

Resolve n: $\begin{cases} c_n = 21-n \\ c_n = 3\text{ cm} \end{cases} \Rightarrow$

$\Rightarrow 21-n=3$

$21-3=n$

$n=18$

Back to D_i now: $\boxed{D_i = c_i - 2}$ $i \geq 2$, $c_1 = 20$, $D_1 = 20$

$D_i = (21-i) - 2$

$D_i = 21-2-i$

$D_i = 19-i$

the 18-th circle has 3cm diameter
$\hat{n}$

the bottom is given by $S_n: D_1, D_2, D_3, \ldots D_n$ (D_n: 1cm, 18th.)
and the D_1 is not exactly based on $D_i = 19-i$ and will need an adjustment
$D_1 = 19-1 = 18$
but instead should be 20
@ the end adjust S_n for the missing 2 units so the bottom will be:

$\begin{cases} D_1: 18\text{ cm} \\ D_2: 17 \\ D_3: 16 \\ \vdots \\ D_{18}: 1\text{ cm} \end{cases}$

later adjustment +2 after S_n is Calculated.

bottom $= S_{18} + 2$

$S_{18} = \frac{n}{2}(D_1 + D_{18}) = \frac{\overset{9}{\cancel{18}}}{\cancel{2}}(18+1) = 9 \times 19 = 171$

$n=18$

bottom $= 171 + 2 = 173$

the adjustment for D_1.

An alternate but similar solution follows

An alternate but similar solution follows

b_i = the new bottom after the addition of a new circle

$b_1 = 20$

⋮

$b_n = b_{n-1} - 2 + c_n$

like before c_n = the diameter of the n-th circle

→ the level of of the 3cm circle remains $n=18$ as calculated in the prev. solution

$c_n = c_{n-1} - 1$

$c_n = 20 + (n-1)(-1)$

based on: "$t_n = t_1 + (n-1)d$"

$c_n = 20 - n + 1$

$\boxed{c_n = 21 - n}$

$b_n = b_{n-1} - 2 + (21 - n)$

$\boxed{b_n = b_{n-1} + 19 - n}$ ← b_n = will give us the final bottom we look for when $n=18$

$b_1 = 20$	$b_1 = 20$
$b_2 = 20 + 19 - 2$	$b_2 = 20 + 1 \cdot 19 - (2)$
$b_3 = 20 + 19 - 2 + 19 - 3$	$b_3 = 20 + 2 \cdot 19 - (2+3)$
$= 20 + 2 \cdot 19 - (2+3)$	
$b_4 = b_3 + 19 - 4$	$b_4 = 20 + 3 \cdot 19 - (2+3+4)$
$b_4 = 20 + 3 \cdot 19 - (2+3+4)$	
$b_5 = b_4 + 19 - \cancel{4}$ -5	$b_5 = 20 + 4 \cdot 19 - (2+3+4+5)$
$b_5 = 20 + 4 \cdot 19 - (2+3+4+5)$	⋮

the following formula emerges:

$b_n = 20 + (n-1) \cdot 19 - (2+3+4+\ldots+n)$

BTW $= \sum_{i=2}^{n} i$

b_{18} = the answer, but 1st calc

$2 + 3 + \ldots n = [1 + 2 + \ldots + n] - 1$

$= S_n - 1$

$= \frac{n}{2}[t_1 + t_n] - 1$

$= \frac{n}{2}(1 + n) - 1$

$= \frac{n(n+1)}{2} - 1$

Resolve b_{18}

2

Resolve b_{18}

$$b_n = 20 + (n-1)19 - \left[\frac{n(n+1)}{2} - 1\right]$$

$$b_n = 20 + 19n - 19 - \frac{n(n+1)}{2} + 1$$

$$b_n = 1 + 19n - \frac{n(n+1)}{2} + 1$$

$$b_n = 19n - \frac{n(n+1)}{2} + 2$$

$$\Rightarrow b_{18} = 19 \cdot 18 - \frac{\cancel{18}^{9}(19)}{\cancel{2}_{1}} + 2$$

$$b_{18} = 342 - 171 + 2$$

$$b_{18} = 171 + 2$$

$$\underline{b_{18} = 173} \text{ cm.} \checkmark$$

th same result as before